Student Name: ______________________________

Student Number: ____________________________

GE 213.3 - Mechanics of Materials

FINAL EXAMINATION

April 26, 2003

Quest. 1:_____
Quest. 2:_____
Quest. 3:_____
Quest. 4:_____
Quest. 5:_____
Quest. 6:_____

Professor: B. Sparling

Time Allowed: 3 Hours

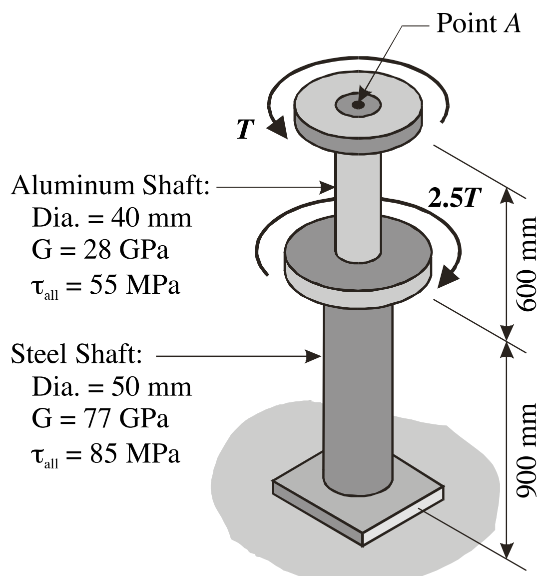
Notes:
- Closed book examination; Calculators may be used
- The value of each question is provided along the left margin
- Supplemental material is provided at the end of the exam (formulas)
- Show **all** your work, including all formulas, calculations and units
- Write your work in the space provided on the examination sheet. (The backs of the examination sheets may also be used if required)

MARKS

15 **QUESTION 1:** The shaft consisting of steel and aluminum segments is subjected to torques of T and $2.5T$, in the locations and directions shown below.

a) Given the allowable shear stresses τ_{all} in the two materials as indicated on the sketch, determine the maximum allowable magnitude of the torque T.

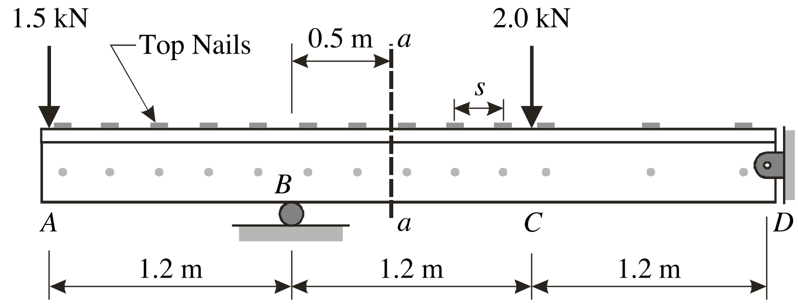
b) If, instead of the value calculated in Part a), the torque $T = 250\ \mathrm{N \cdot m}$, calculate the angle of twist at Point A on top of the shaft.

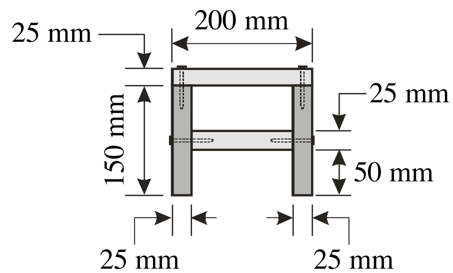


17 **QUESTION 2:** A timber beam is constructed from four boards and is supported and loaded as shown. Section *a-a* is located 0.5 m to the right of the support at Point *B*.

a) Determine the maximum **tensile** normal stress at Section *a-a*.

b) If each nail can safely support a shear force of 300 N, determine the maximum allowable spacing "*s*" of nails **on the top** of the beam at Section *a-a*.



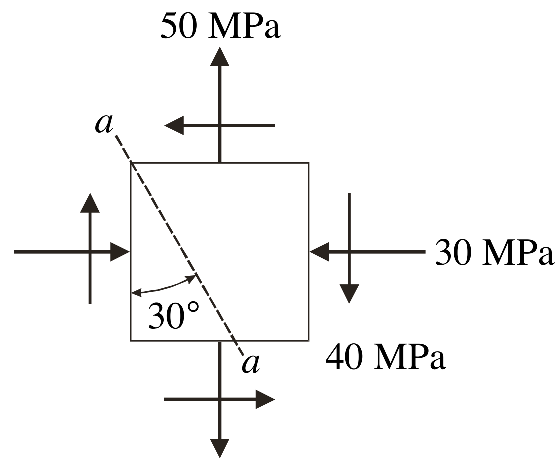


Section *a-a*

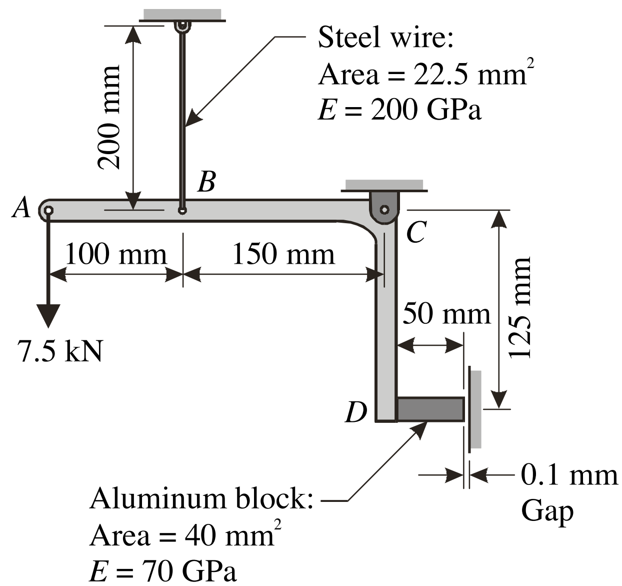
16 **QUESTION 3:** The state of stress at a given point on the surface of an object is illustrated below. Using the Mohr's circle approach, determine the following stresses and indicate their orientations on a sketch:

a) The principal normal stresses; and

b) The normal and shear stresses on Section *a-a*.



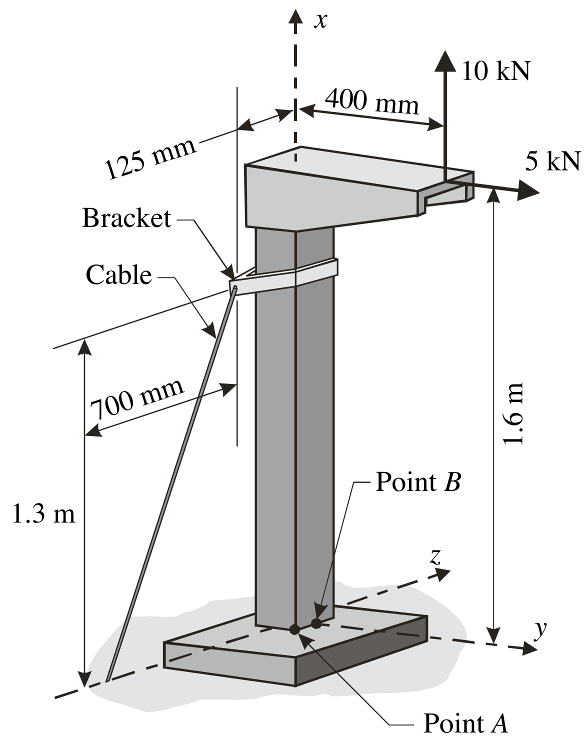
19 **QUESTION 4:** A 50 mm long aluminum block is attached to the perfectly rigid Link *ABCD* at Point *D*; when the system is unloaded, a 0.1 mm gap exists between this block and the rigid support on its right end. If a 7.5 kN vertical load is applied at Point *A* as shown, calculate the resulting force in the 200 mm long vertical steel wire attached at Point *B*. [Hint: The aluminium block comes into contact with the rigid support at its right end after the load is applied.]

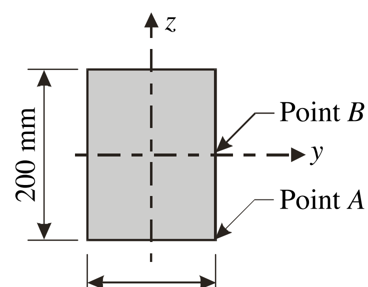


18 **QUESTION 5:** A solid rectangular post supports a platform loaded by a 10 kN vertical force and a 5 kN horizontal force, both of which lie in the *x*-*y* plane. A cable pre-tensioned to a tension of 15 kN lies in the *x*-*z* plane and is attached to a bracket on the post. Determine the following:

a) The vertical normal stress at Point *A*; and

b) The horizontal shear stress at Point *B*.

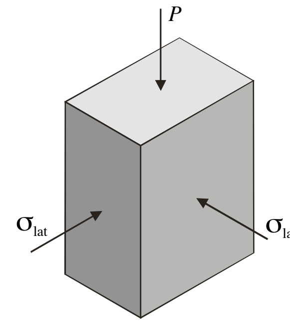




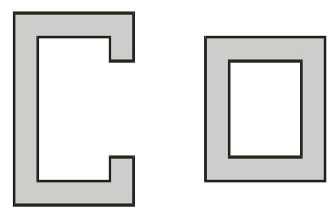
Post Cross-Section
(Looking downward)

<u>QUESTION 6:</u> Provide brief answers to the following questions – answers in point form are acceptable. Diagrams should be used to supplement your responses where appropriate.

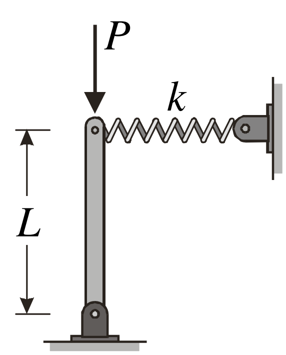
4 **a)** Describe and explain the effect that the lateral normal stresses σ_{lat} applied to the vertical sides of the block shown below will have on the vertical deformations due to the axial normal force *P*.



5 **b)** Compare the relative torsional resistance capacities of the two cross-sectional shapes shown below and explain the differences in terms of the physical mechanisms contributing to the torsional resistance. Assume that both shapes possess similar cross-sectional areas.



6 **c)** In qualitative terms (i.e., without the use of equations), list and explain the possible states of static equilibrium for the axially loaded bar shown below.



- **Normal Stress:** $\sigma_{avg} = \dfrac{P}{A}$ $\quad P = \int_A \sigma \, dA$ • **Bearing Stress:** $\sigma_b = \dfrac{P}{t\,d}$
- **Direct Shear:** $\tau_{avg} = \dfrac{V}{A}$ (Single) or $\tau_{avg} = \dfrac{V}{2A}$ (Double) • **Hooke's Law:** $\sigma = E\,\varepsilon$
- **Allowable Stress:** $F.S. = \dfrac{P_U}{P_D}$ or $F.S. = \dfrac{\sigma_U}{\sigma_D}$ **;** $\sigma_{all} = \dfrac{\sigma_U}{F.S.}$ $\quad P_{all} = \sigma_{all}\,A$ $\quad A_{req} = \dfrac{P_D}{\sigma_{all}}$
- **Stresses on Oblique Planes:** $\sigma_\theta = \dfrac{P\cos\theta}{A_o/\cos\theta} = \dfrac{P}{A_o}\cos^2\theta$ **;** $\tau_\theta = \dfrac{P\sin\theta}{A_o/\cos\theta} = \dfrac{P}{A_o}\sin\theta\cos\theta$
- **Average Normal Strain:** $\varepsilon = \dfrac{\delta}{L_o} = \dfrac{L^* - L}{L}$ • **Poisson's Ratio:** $\varepsilon_y = \varepsilon_z = -\nu\,\varepsilon_x$
- **Axial Deformations:** $\delta = \dfrac{P\,L_o}{A_o\,E}$**;** $\delta_{tot} = \sum_i \dfrac{P_i\,L_i}{A_i\,E_i}$**;** $\delta = \int_0^L \dfrac{P(x)}{A(x)\,E(x)}\,dx$
- **General Hooke's Law:** $\varepsilon_x = \dfrac{\sigma_x}{E} - \nu\dfrac{\sigma_y}{E} - \nu\dfrac{\sigma_z}{E}$**;** $\varepsilon_y = -\nu\dfrac{\sigma_x}{E} + \dfrac{\sigma_y}{E} - \nu\dfrac{\sigma_z}{E}$**;** $\varepsilon_z = -\nu\dfrac{\sigma_x}{E} - \nu\dfrac{\sigma_y}{E} + \dfrac{\sigma_z}{E}$
- **Shearing Strain & Stress:** $\theta^* = \dfrac{\pi}{2} - \gamma_{xy}$**;** $\gamma_{xy} = \dfrac{\tau_{xy}}{G}$; $\gamma_{yz} = \dfrac{\tau_{yz}}{G}$; $\gamma_{zx} = \dfrac{\tau_{zx}}{G}$**;** $G = \dfrac{E}{2(1+\nu)}$
- **Thermal Deformations:** $\delta_T = \alpha\,(\Delta T)\,L_o$ **;** $\varepsilon_T = \dfrac{\delta_T}{L_o}$ • **Resultant Torque:** $T = \int_A \rho\,\tau\,dA$
- **Torsional Strains:** $\gamma = \dfrac{\rho\,\phi}{L}$ **;** $\gamma_{\max} = \dfrac{c\,\phi}{L}$ **;** $\gamma = \left(\dfrac{\rho}{c}\right)\gamma_{\max}$
- **Torsional Stresses:** $\tau = \left(\dfrac{\rho}{c}\right)\tau_{\max}$ $\quad \tau_{\max} = \dfrac{T\,c}{J}$ $\quad \tau = \dfrac{T\,\rho}{J}$ $\quad J = \int_A \rho^2\,dA = \dfrac{\pi}{2}c^4$
- **Torsional Angle of Twist:** $\phi = \dfrac{T\,L}{J\,G}$ • **Torsion - Gear Compatibility:** $\phi_1\,\rho_1 = \phi_1\,\rho_2$
- **Pure Bending - Normal Strain:** $\varepsilon_x = -\dfrac{y}{\rho}$ $\quad \varepsilon_{\max} = c/\rho$ $\quad \varepsilon_x = -\dfrac{y}{c}\varepsilon_m$
- **Pure Bending - Normal Stress:** $\sigma_x = -\dfrac{y}{c}\sigma_m$ $\quad \sigma_x(y) = -\dfrac{M\,y}{I}$ $\quad \sigma_{\max} = \dfrac{M\,c}{I}$
- **Section Properties:** $I = \int_A y^2\,dA$**;** $I = \sum_i \left(I_i + A_i\,d_i^2\right)$; Centroid: $\int_A y\,dA = 0$; $\bar{y}\,A = \sum_i y_i\,A_i$
- **Biaxial Bending:** $\sigma_x = -\dfrac{M_z\,y}{I_z} + \dfrac{M_y\,z}{I_y}$; $\tan\phi = \dfrac{I_z}{I_y}\tan\theta$; $M_z = M\cos\theta$; $M_y = M\sin\theta$
- **Eccentric Axial Loading:** $\sigma_x = \dfrac{P}{A} - \dfrac{M\,y}{I}$; • **Shear Flow:** $q = V\,Q/I$
- **Flexural Shear Stress:** $\tau_{\text{ave}} = \dfrac{V\,Q}{I\,t}$; $Q = \int_A y\,dA = A\,\bar{y}$ • **Discrete Fasteners:** $F_N = q \times s$
- **Plane Stress Transformations:** $\sigma_{x'} = \dfrac{\sigma_x + \sigma_y}{2} + \dfrac{\sigma_x - \sigma_y}{2}\cos(2\theta) + \tau_{xy}\sin(2\theta)$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos(2\theta) - \tau_{xy}\sin(2\theta)\ ; \quad \tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin(2\theta) + \tau_{xy}\cos(2\theta)$$